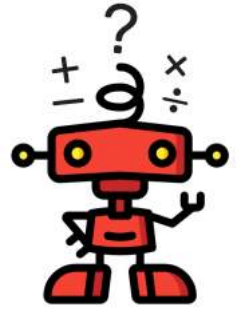


HELPFUL HINTS

FOR SOLVING MASHUP MATH PUZZLES



Page	Helpful Hint
2	Intro: Symbols Representing Numbers
3	Multiple Symbols Can Have the Same Value
4	Advanced Substitution
5	Following the Order of Operations
6	Double Symbols
7	Parentheses and Square Roots
8	Negative Numbers
9	Place Value and Decimals
10	Multiplication Tables
11	Area Models
12-13	Which One Doesn't Belong?
14-15	Think-Notice-Wonder Writing Prompts
16	Two Truths and One Lie!
17	It's Ok to Make Mistakes! Here's Why
18	Use GEMS (Not PEMDAS)
19	How to Print

HELPFUL HINT #1

Symbols represent numbers.

Example A

If $2 + \text{star} = 9$

then $\text{star} = 7$

Example B

If $\text{rabbit} + \text{rabbit} = 8$



then $\text{rabbit} = 4$

HELPFUL HINT #2

Multiple symbols can have the same value.

Example

If  = 

and  +  = 6.

then  &  both equal 3.



HELPFUL HINT #3

Advanced Substitution

(Sometimes two symbols being equal to each other isn't always obvious)


Example A

If  -  = 0,

then  = .

because any number minus itself equals zero.

Example B

If  ÷  = 1

then  = .

because any non-zero number divided by itself equals one.

* You can assume that  does not equal zero.

HELPFUL HINT #4

Remember to follow the
Order of Operations

If  = 8  = 4  = 2

Example A

Perform multiplication/division before
addition/subtraction.

$$\begin{array}{r} \text{stapler} - \text{lamp} \times \text{tape} \\ \text{8} - 2 \times 4 \\ \text{8} - 8 = 0 \end{array}$$

Example B

Perform multiplication/division from left to right:

$$\begin{array}{r} \text{stapler} \div \text{tape} \times \text{lamp} \\ \text{8} \div 4 \times \text{lamp} \\ 2 \times 2 = 4 \end{array}$$

HELPFUL HINT #5

Double-Symbols

A double-symbol is a grouping that can be interpreted as follows:

$$\begin{array}{c} \text{🌮} \text{🌮} \\ \text{🌮} \end{array} = \left(\begin{array}{c} \text{🌮} \\ \text{🌮} \end{array} + \begin{array}{c} \text{🌮} \\ \text{🌮} \end{array} \right)$$

or

$$\begin{array}{c} \text{🌮} \text{🌮} \\ \text{🌮} \end{array} = \left(\begin{array}{c} \text{🌮} \\ \text{🌮} \end{array} \times 2 \right)$$

Example

If $\begin{array}{c} \text{🎮} \text{🎮} \\ \text{🎮} \end{array} - 5 = 13$,

then $\begin{array}{c} \text{🎮} \\ \text{🎮} \end{array} = 9$

because $\left(\begin{array}{c} \text{🎮} \\ \text{🎮} \end{array} \times 2 \right) - 5 = 13$
 $18 - 5 = 13$

HELPFUL HINT #6

Parenthesis and Square Roots

$$\text{🎃} = 25 \quad \text{👻} = 9 \quad \text{🧙} = 3$$

EXAMPLE A

When following the order of operations, always perform groupings first!

$$\begin{aligned} & \left(\text{🎃} - \text{👻} \right) \times \text{🧙} \\ & \quad \underbrace{\hspace{2cm}} \\ & \quad 25 - 9 \quad \times \text{🧙} \\ & \quad 16 \quad \times 3 = 48 \end{aligned}$$

EXAMPLE B

Perfect Squares and Square Roots Review:

$$\sqrt{49} = 7 \quad \text{because } 7 \times 7 = 49$$

$$\sqrt{\text{🎃}} = 5 \quad \text{because } 5 \times 5 = \text{🎃}$$

$$\sqrt{\text{👻}} = \text{🧙} \quad \text{because } \text{🧙} \times \text{🧙} = \text{👻}$$

HELPFUL HINT #7

Negative Numbers

$$\text{lemon} = 22$$

$$\text{orange} = -5$$

$$\text{cherries} = -10$$

Examples:

Adding a negative is the same as subtracting.

$$\text{lemon} + \text{cherries} \Rightarrow 22 + -10 \Rightarrow 22 - 10 = 12$$

Subtracting a negative is the same as adding.

$$\text{orange} - \text{cherries} \Rightarrow -5 - -10 \Rightarrow -5 + 10 = 5$$

double negative ↑

A negative times a positive is negative.

$$\text{lemon} \times \text{orange} \Rightarrow 22 \times -5 = -110$$

A negative times a negative is positive.

$$\text{orange} \times \text{cherries} \Rightarrow -5 \times -10 = 50$$

HELPFUL HINT #8

Place Value and Decimals

Place Value Chart for Decimals

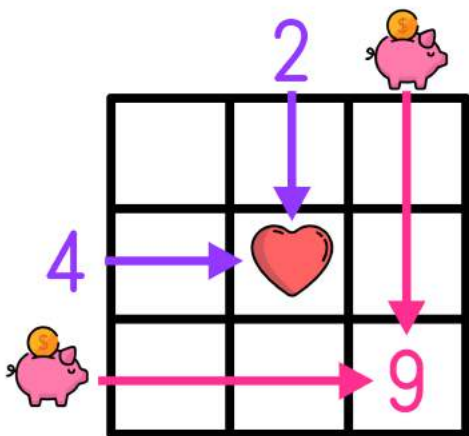
T H O U S A N D S	H U N D R E D S	T E N S	O N E S	D E C I M A L	T E N T H S	H U N D R E D T H S	T H O U S A N D T H S
		3	2	•	0	4	

$$32.04 + 8.21 \Rightarrow \begin{array}{r} 32.04 \\ + 8.21 \\ \hline = 40.25 \end{array}$$

HELPFUL HINT #9

Multiplication Tables

Multiplication tables work like a Bingo Board where each box represents the product of its corresponding column and row.



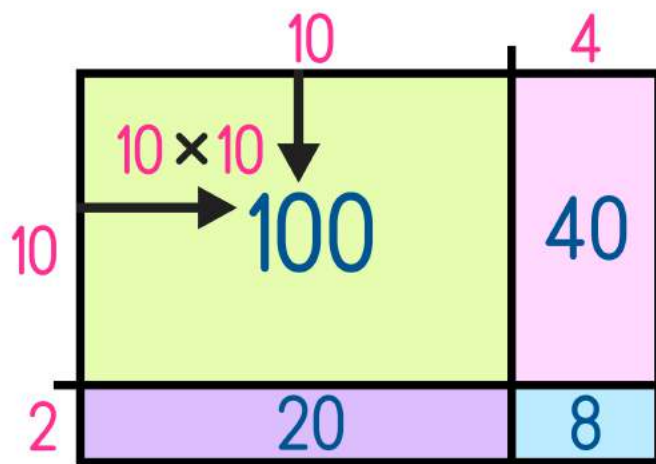
$$\text{♥} = 2 \times 4 \rightarrow \text{♥} = 8$$

$$9 = \text{piggy bank} \times \text{piggy bank} \rightarrow \text{piggy bank} = 3$$

HELPFUL HINT #10

Area Models

Area Models work like a Bingo Board where each box represents the product of its corresponding column and row and the sum of all four inner boxes represents the total.



$$14 \times 12 = 168$$

$$\begin{array}{r} \rightarrow 100 \\ 40 \\ 20 \\ + 8 \\ \hline 168 \end{array}$$

HELPFUL HINT #11

Which One Doesn't Belong?

If you are looking to make your math warm-ups more visual and thought-provoking, then starting your lessons with *Which One Doesn't Belong?* (*WODB*) activities is a great strategy for instantly sparking creative and critical student thinking that will last for the entire lesson.

How do *WODB* activities work?

Instead of working on practice problems during a lesson's warm-up, students will observe and reflect upon a graphic displaying four images. They will then apply their mathematical and reasoning skills to decide which of the four items does not belong and also justify why their choice is valid.

Are *WODB* activities like visual multiple choice questions?

Nope. *WODB* activities do not have a single correct answer. These graphics are designed to be interpreted in a variety of different ways in order to spark deep mathematical thinking and discussion (in small groups, whole class, or both).

Here's an Example:

Consider the graphic below and the different responses by Students A, B, and C.

<u>Student A</u>	<u>Student B</u>	<u>Student C</u>												
<p>Which one doesn't belong?</p> <table border="1"><tr><td>8</td><td>27</td></tr><tr><td>64</td><td>16</td></tr></table>	8	27	64	16	<p>Which one doesn't belong?</p> <table border="1"><tr><td>8</td><td>27</td></tr><tr><td>64</td><td>16</td></tr></table>	8	27	64	16	<p>Which one doesn't belong?</p> <table border="1"><tr><td>8</td><td>27</td></tr><tr><td>64</td><td>16</td></tr></table>	8	27	64	16
8	27													
64	16													
8	27													
64	16													
8	27													
64	16													

Student A says: *I think 27 doesn't belong because it is the only value that is not divisible by four. Also, 27 is the only odd number in the group.*

Student B says: *I think 64 doesn't belong because all of the other numbers are either a perfect square or a perfect cube, but 64 is both!*

Student C says: *I think 16 doesn't belong because it is not a perfect cube and the other three are.*

Notice that all three students have engaged in deep mathematical thinking and their curiosity and interest will carry on throughout the day's lesson!

As teacher, you can steer this discussion in a variety of directions by asking follow-up questions like:

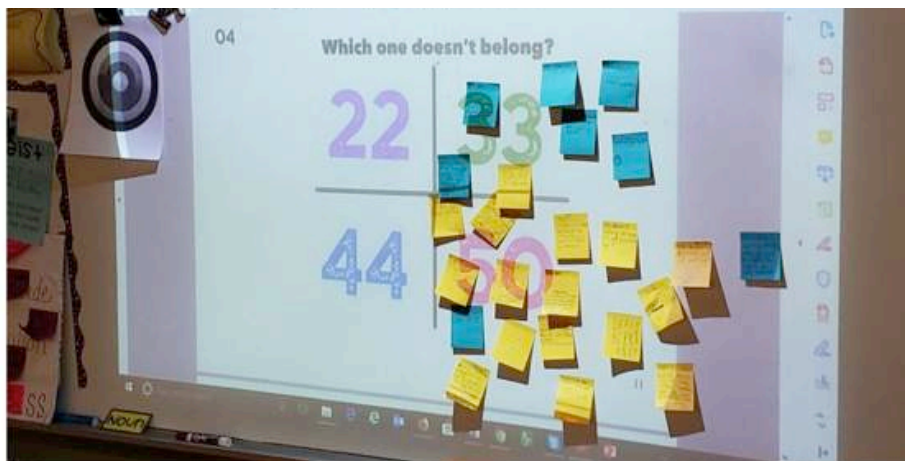
What justification could you use to say that 8 doesn't belong?

What other justifications could student A have used to decide that 27 does not belong?

How can students A, B, and C all be correct even though they each chose different values?

What topics and grade levels are WODB activities best suited for?

WODB activities can be used for all grade levels and topics. The graphics can be topic/lesson specific or more broad and open-ended. Remember, the idea is to spark enough student thinking, interest, and curiosity at the beginning of your lesson to last for the entire class!



Using post-it notes is a great strategy for assessing student thinking, especially when using WODB as an exit ticket.

HELPFUL HINT #12

Think-Notice-Wonder Writing Prompts

Writing about math helps kids to organize their thinking, use key vocabulary, and communicate mathematically—which leads to deep and meaningful understanding.

Over the past few years, math teachers are incorporating more writing activities into their lesson plans—a trend that is being driven by the use of highly engaging *think-notice-wonder* writing prompts that spark deep mathematical discussion and are highly effective as warm-up or cool-down activities.

This strategy has recently been endorsed by the [National Council of Teachers of Mathematics](#):

By asking ***What do you notice? What do you wonder?*** we give students opportunities to see problems in big-picture ways, and discover multiple strategies for tackling a problem. Self-confidence, reflective skills, and engagement soar, and students discover that the goal is not to be "over and done," but to realize the many different ways to approach problems.

How does it work?

Math teachers often struggle to find topics for their kids to write about. Sometimes the best way to encourage creativity and exploration is simply posting an image and asking students to describe what they *think, notice, and wonder* about what they are seeing.

The best way to use *think-notice-wonder* activities is to choose an image every day and project it as large as you can at the front of your classroom.

Then, have students write 3 sentences about the image starting with:

- **I think...** **I notice...** **I Wonder...**

- ✓ You may want to have students share their entries in a *daily math journal*. This practice will get them used to writing about mathematics regularly.
- ✓ Try not to give too many prompts. You'll be surprised by how creative and detailed student responses will become over time!
- ✓ Try to choose images that work with the day's topic/theme

What would *think-notice-wonder* look like in YOUR classroom?

Imagine an unusual day where your kids enter class expecting a normal warm-up practice problem but are caught by surprise.



They see the above image of a drone delivering a pizza displayed on the board along with a writing prompt that calls for them to complete the statements *I think...*, *I notice...*, and *I wonder...*

What kind of creative thoughts and ideas would they have?

How much weight can the drone carry? How many pizzas can it hold at once?

How would thinking about this image activate their prior knowledge and spark their curiosity?

What are the dimensions of the largest and smallest pizza box it can hold? Does it have to be square?

What kind of anticipation would it build for an upcoming lesson or activity?

Does the drone use GPS coordinates to get from point to point?

Why *Think-Notice-Wonder*?

Engaging in *think-notice-wonder* writing activities at the start of a math class is a great way to ignite student thinking, spark creativity, and build anticipation.

Even if students are not directly engaged in mathematical problem-solving, their curiosity and interest will carry on throughout the day's lesson.

Be mindful that your kids will need some time to get used to these kinds of activities, but after a week or so, you'll be pleasantly surprised by the spike in engagement, boost in student enthusiasm and high quality of responses!

HELPFUL HINT #13

Two Truths and One Lie!

Capturing your students interest and curiosity during the first few minutes of class is the key to keeping them engaged for your entire lesson. But not all math warm up activities are created equally.

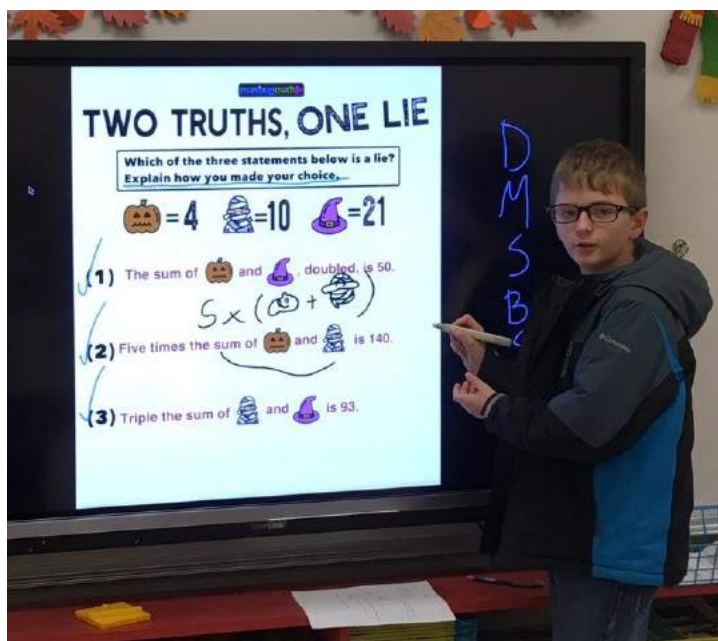
Math teachers miss out on activating their students' critical thinking and reasoning skills when they assign routine, lower-level practice problems during the first five minutes of class.

However, when you use the right mix of fun and though-provoking math warm up activities to start your lessons, student engagement spikes, as your kids will constantly be wondering about what is coming next.

Two Truths and One Lie!

I recently started using ***Two Truths and One Lie (2T1L)*** activities, where students are presented with three mathematical statements (only two of which are true) and they have to identify which statement is a lie and justify why their choice is correct. The results? Pretty amazing. 2T1L taught me that my students love to argue and state their case (in small groups or to the whole class).

In short, 2T1L is a fun way to spark deep mathematical thinking and open discussion at the start (or end—2T1L activities make great exit tickets) of any lesson.

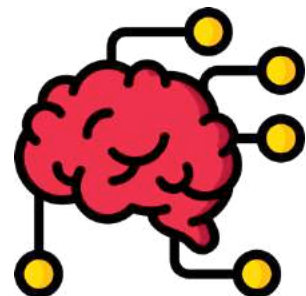


HELPFUL HINT #14

It's okay to make mistakes!

Here are some tips to help you persevere through challenging problems:

- Read each puzzle carefully and think about the problem for a while before doing anything.
- Utilize strategies such as visualizing, drawing diagrams, and trial-and-error when you don't know where to start.
- Don't get discouraged! When you are struggling and making mistakes, you are in the process of learning. This is called having a growth mindset!
- w Whenever you find a solution, ask yourself "does my answer make sense?"
- If you are stuck on a problem, close the book, take a short break, and do something else like taking a short walk. You'll be surprised by how the problem will become more manageable when you return.
- Have fun!



HELPFUL HINT #15

Review: The GEMS Method for Performing Order of Operations

mashupmath



Order of Operations



G

Groupings

$() \{ \} []$

E

Exponents

n^2

M

Multiply/Divide

Left to Right

$\div / \times \cdot$

S

Subtract/Add

Left to Right

$+ -$

EXAMPLE

$$(5 \times 8) \div 5 \times 2 - 1 + 3$$

STEP ONE

GEMS

$$(5 \times 8) \div 5 \times 2 - 1 + 3$$

Groupings First

STEP TWO

GEMS

$$40 \div 5 \times 2 - 1 + 3$$

There are no exponents in this example!

STEP THREE

GEMS

$$40 \div 5 \times 2 - 1 + 3$$

Multiply and Divide from left to right.

STEP FOUR

GEMS

$$16 - 1 + 3$$

Subtract and Add from left to right.

The answer is **18**

HELPFUL HINT #16

How to Print (PDF VERSION ONLY)

Starting with the PDF open on your computer:

- 1.) Select the page that you want to print.
- 2.) Right-click on the page and select *PRINT*.
- 3.) Select the number of copies you want.

